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Lightlike p-Branes: Mass "Inflation" and Lightlike Braneworlds

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Abstract

Lagrangian description of lightlike p-branes is presented in two equivalent forms – a Polyakov-type formulation and a dual to it Nambu-Goto-type formulation. Next, the properties of lightlike brane dynamics in generic gravitational backgrounds of spherically symmetric and axially symmetric type are discussed in some detail: "horizon straddling" and "mass inflation" effects for codimension-one lightlike branes and ground state behavior of codimension-two lightlike "braneworlds".

1. Introduction

Lightlike branes (*LL-branes*, for short) are of particular interest in general relativity primarily due to their role in the effective treatment of many cosmological and astrophysical effects: (i) impulsive lightlike signals arising in cataclysmic astrophysical events [1]; (ii) the "membrane paradigm" theory of black hole physics [2]; (iii) thin-wall description of domain walls coupled to gravity [3, 4]. More recently *LL-branes* became significant also in the context of modern non-perturbative string theory [5].

Here we first present explicit reparametrization invariant $(p+1)$-dimensional world-volume actions describing *LL-brane* dynamics in two equivalent forms: (i) Polyakov-type formulation, and (ii) Nambu-Goto-type formulation dual to the first one.

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Unlike ordinary Nambu-Goto p-branes (describing massive brane modes) our models yield intrinsically lightlike p-branes (the induced metric becoming singular on-shell) with the additional crucial property of the *brane tension* appearing as a *non-trivial dynamical degree of freedom.* The latter characteristic feature significantly distinguishes our lightlike p-brane models from the previously proposed *tensionless* p-branes (for a review, see *e.g.* [6]) which rather resemble a p-dimensional continuous distribution of massless point-particles.

Next we discuss the properties of *LL-brane* dynamics in generic gravitational backgrounds. The case with two extra dimensions (codimension-two *LL-branes*) is studied from the point of view of "braneworld" scenarios. Unlike conventional braneworlds, where the underlying branes are of Nambu-Goto type and in their ground state they position themselves at some fixed point in the extra dimensions of the bulk space-time, our lightlike braneworlds perform in the ground state non-trivial motions in the extra dimensions – planar circular, spiral winding *etc* depending on the topology of the extra dimensions.

The special case of codimension-one *LL-branes* is qualitatively different. Here the *LL-brane* dynamics dictates that the bulk space-time with a bulk metric of spherically or axially symmetric type must possess an event horizon which is automatically occupied by the *LL-brane* ("horizon straddling"). We study several cases of "horizon straddling" solutions. In the case of Kerr "horizon straddling" by a *LL-brane* there is the additional effect of brane rotation "dragged" by the Kerr black hole.

For the inner Reissner-Nordström horizon we find a time symmetric "mass inflation" effect, which also holds for de Sitter horizon. In this case the dynamical tension of the *LL-brane* blows up as time approaches $\pm\infty$ due to its exponential quadratic time dependence. For the Schwarzschild and the outer Reissner-Nordström horizons, on the other hand, we obtain "mass deflationary" scenarios where the dynamical *LL-brane* tension vanishes at large positive or large negative times. Another set of solutions with asymmetric (w.r.t. $t \to -t$) exponential linear time dependence of the *LL-brane* tension also exists. By fine tuning one can obtain a constant time-independent brane tension as a special case. The latter holds in particular for *LL-branes* moving in extremal Reissner-Nordström or maximally rotating Kerr black hole backgrounds.

2. World-Volume Actions for Lightlike Branes

In refs.[7, 8, 9] we have proposed the following generalized Polyakov-type formulation of the Lagrangian dynamics of *LL-branes* in terms of the world-volume action:

$$S = \int d^{p+1}\sigma\, \Phi(\varphi) \left[-\frac{1}{2}\gamma^{ab} g_{ab} + L\big(F^2\big) \right] . \tag{1}$$

Here γ_{ab} denotes the intrinsic Riemannian metric on the world-volume;

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \tag{2}$$

is the induced metric (the latter becomes *singular* on-shell – lightlikeness, cf. second Eq.(7) below);

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{I_1\ldots I_{p+1}}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\varphi^{I_1}\ldots\partial_{a_{p+1}}\varphi^{I_{p+1}} \tag{3}$$

is an alternative non-Riemannian reparametrization-covariant integration measure density replacing the standard $\sqrt{-\gamma} \equiv \sqrt{-\det\|\gamma_{ab}\|}$ and built from auxiliary world-volume scalars $\{\varphi^I\}_{I=1}^{p+1}$;

$$F_{a_1\ldots a_p} = p\partial_{[a_1}A_{a_2\ldots a_p]} \quad , \quad F^{*a} = \frac{1}{p!}\frac{\varepsilon^{aa_1\ldots a_p}}{\sqrt{-\gamma}}F_{a_1\ldots a_p} \tag{4}$$

are the field-strength and its dual one of an auxiliary world-volume $(p-1)$-rank antisymmetric tensor gauge field $A_{a_1\ldots a_{p-1}}$ with Lagrangian $L(F^2)$ $(F^2 \equiv F_{a_1\ldots a_p}F_{b_1\ldots b_p}\gamma^{a_1b_1}\ldots\gamma^{a_pb_p})$.

Equivalently one can rewrite (1) as:

$$S = \int d^{p+1}\sigma\,\chi\sqrt{-\gamma}\left[-\frac{1}{2}\gamma^{ab}g_{ab} + L\big(F^2\big)\right] \quad , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \tag{5}$$

The composite field χ plays the role of a *dynamical (variable) brane tension.*

For the special choice $L\big(F^2\big) = \big(F^2\big)^{1/p}$ the above action becomes invariant under Weyl (conformal) symmetry:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\,\gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \tag{6}$$

with Jacobian $\det\left\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\right\| = \rho$.

Consider now the equations of motion corresponding to (1) w.r.t. φ^I and γ^{ab}:

$$\frac{1}{2}\gamma^{cd}g_{cd} - L(F^2) = M \quad , \quad \frac{1}{2}g_{ab} - F^2L'(F^2)\left[\gamma_{ab} - \frac{F^*_aF^*_b}{F^{*\,2}}\right] = 0\;. \tag{7}$$

Here M is an integration constant and $F^{*\,a}$ is the dual field strength (4). Both Eqs.(7) imply the constraint $L\big(F^2\big) - pF^2L'\big(F^2\big) + M = 0$, *i.e.*

$$F^2 = F^2(M) = \text{const on} - \text{shell}\;. \tag{8}$$

The second Eq.(7) exhibits *on-shell singularity* of the induced metric (2):

$$g_{ab}F^{*\,b} = 0\;. \tag{9}$$

Further, the equations of motion w.r.t. world-volume gauge field $A_{a_1 \ldots a_{p-1}}$ (with χ as defined in (5) and accounting for the constraint (8)):

$$\partial_{[a}\left(F^{*}_{b]}\,\chi\right) = 0 \tag{10}$$

allow us to introduce the dual "gauge" potential u:

$$F^{*}_{a} = \mathrm{const}\,\frac{1}{\chi}\partial_a u \;. \tag{11}$$

We can rewrite second Eq.(7) (the lightlike constraint) in terms of the dual potential u as:

$$\gamma_{ab} = \frac{1}{2a_0}\,g_{ab} - \frac{2}{\chi^2}\,\partial_a u \partial_b u \quad , \quad a_0 \equiv F^2 L'\left(F^2\right)|_{F^2=F^2(M)} = \mathrm{const} \tag{12}$$

($L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2). From (11) and (8) we have the relation:

$$\chi^2 = -2\gamma^{ab}\partial_a u \partial_b u \;, \tag{13}$$

and the Bianchi identity $\partial_a F^{*\,a} = 0$ becomes:

$$\partial_a\Big(\frac{1}{\chi}\sqrt{-\gamma}\gamma^{ab}\partial_b u\Big) = 0 \;. \tag{14}$$

Finally, the X^μ equations of motion produced by the (1) read:

$$\partial_a\left(\chi\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^{\mu}_{\nu\lambda}(X) = 0 \tag{15}$$

where $\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the Christoffel connection for the external metric.

It is now straightforward to prove that the system of equations (13)–(15) for (X^μ, u, χ), which are equivalent to the equations of motion (7)–(10),(15) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu-Goto-type world-volume action:

$$S_{\mathrm{NG}} = -\int d^{p+1}\sigma\, T\sqrt{-\det\|g_{ab} - \frac{1}{T^2}\partial_a u \partial_b u\|} \;. \tag{16}$$

Here g_{ab} is the induced metric (2); T is *dynamical* tension simply related to the dynamical tension χ from the Polyakov-type formulation (5) as $T^2 = \frac{\chi^2}{4a_0}$ with a_0 – same constant as in (12).

Henceforth we will consider the initial Polyakov-type form (1) of the *LL-brane* world-volume action. Invariance under world-volume reparametrizations allows to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \;\; (i = 1, \ldots, p) \; , \; \gamma^{00} = -1 \tag{17}$$

Also, in what follows we will use a natural ansatz for the "electric" part of the auxiliary world-volume gauge field-strength:

$$F^{*i} = 0 \;\; (i = 1, \ldots, p) \quad , \quad \text{i.e. } \; F_{0 i_1 \ldots i_{p-1}} = 0 \; , \tag{18}$$

The Bianchi identity ($\partial_a F^{*\,a} = 0$) together with (17)–(18) and the definition for the dual field-strength in (4) imply:

$$\partial_0 \gamma^{(p)} = 0 \quad \text{where} \;\; \gamma^{(p)} \equiv \det \|\gamma_{ij}\| \; . \tag{19}$$

Then *LL-brane* equations of motion acquire the form (recall definition of g_{ab} (2)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0 \quad , \quad g_{0i} = 0 \quad , \quad g_{ij} - 2a_0\, \gamma_{ij} = 0 \tag{20}$$

(Virasoro-like constraints), where the M-dependent constant a_0 (the same as in (12)) must be strictly positive;

$$\partial_i \chi = 0 \qquad \text{(remnant of Eq.(10))} \; ; \tag{21}$$

$$-\sqrt{\gamma^{(p)}}\partial_0 \left(\chi \partial_0 X^\mu\right) + \partial_i \left(\chi \sqrt{\gamma^{(p)}} \gamma^{ij} \partial_j X^\mu\right)$$
$$+\chi \sqrt{\gamma^{(p)}} \left(-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda\right) \Gamma^\mu_{\nu\lambda} = 0 \; . \tag{22}$$

3. Lightlike Branes in Gravitational Backgrounds: Codimension-Two

Let us split the bulk space-time coordinates as:

$$(X^\mu) = (x^a, y^\alpha) \equiv \left(x^0 \equiv t, x^i, y^\alpha\right) \tag{23}$$
$$a = 0, 1, \ldots, p \;\; , \;\; i = 1, \ldots, p \;\; , \;\; \alpha = 1, \ldots, D - (p+1)$$

and consider background metrics $G_{\mu\nu}$ of the form:

$$ds^2 = -A(t,y)(dt)^2 + C(t,y) h_{ij}(\vec{x}) dx^i dx^j + B_{\alpha\beta}(t,y) dy^\alpha dy^\beta \tag{24}$$

Here we will discuss the simplest non-trivial ansatz for the *LL-brane* embedding coordinates:

$$X^a \equiv x^a = \sigma^a \quad , \quad X^{p+\alpha} \equiv y^\alpha = y^\alpha(\tau) \quad , \quad \tau \equiv \sigma^0 \; . \tag{25}$$

and will take the particular solution $\chi = \text{const}$ of Eq.(21) for the dynamical tension (for more general time-dependent dynamical tension solutions, see next Section).

Now the *LL-brane* (gauge-fixed) equations of motion (19)–(22) describing its dynamics in the extra dimensions reduce to:

$$\dot{y}^{\alpha}\frac{\partial}{\partial y^{\alpha}}A\,|_{y=y(\tau)}=0 \quad , \quad \dot{y}^{\alpha}\frac{\partial}{\partial y^{\alpha}}C\,|_{y=y(\tau)}=0 \; , \tag{26}$$

$$-A(y(\tau)) + B_{\alpha\beta}(y(\tau))\,\dot{y}^{\alpha}\dot{y}^{\beta}=0 \; , \tag{27}$$

$$\ddot{y}^{\alpha} + \dot{y}^{\beta}\dot{y}^{\gamma}\,\Gamma^{\alpha}_{\beta\gamma} + B^{\alpha\beta}\left(\frac{p\,a_0}{C(y)}\frac{\partial}{\partial y^{\beta}}C(y) + \frac{1}{2}\frac{\partial}{\partial y^{\beta}}A(y)\right)|_{y=y(\tau)}=0 \tag{28}$$

(recall $a_0 = \text{const}$ as in (12)).

Example 1: Two Flat Extra Dimensions. In this case:

$$y^{\alpha}=(\rho,\phi) \quad , \quad B_{\alpha\beta}(y)dy^{\alpha}dy^{\beta}=d\rho^2+\rho^2 d\phi^2 \; ; \tag{29}$$

$$A=A(\rho) \;\; , \;\; C=C(\rho) \quad ; \;\; \rho=\rho_0=\text{const} \;\; ; \;\; \phi(\tau)=\omega\tau \; , \tag{30}$$

where:

$$\omega^2=\frac{A(\rho_0)}{\rho_0^2} \quad , \quad A(\rho_0)=\rho_0\left(\frac{p\,a_0}{C(\rho)}\partial_{\rho}C+\frac{1}{2}\partial_{\rho}A)\right)|_{\rho=\rho_0} \; . \tag{31}$$

Thus, we find that the *LL-brane* performs a *planar circular motion* in the flat extra dimensions whose radius ρ_0 and angular velocity ω are determined from (31). This property of the *LL-branes* has to be contrasted with the usual case of Nambu-Goto-type braneworlds which (in the ground state) occupy a *fixed position* in the extra dimensions.

Example 2: Toroidal Extra Dimensions. In this case:

$$y^{\alpha}=(\theta,\phi) \;\; , \;\; 0\leq\theta,\phi\leq 2\pi \quad , \quad B_{\alpha\beta}(y)dy^{\alpha}dy^{\beta}=d\theta^2+a^2 d\phi^2 \tag{32}$$

The solutions read:

$$\theta(\tau)=\omega_1\tau \quad , \quad \phi(\tau)=\omega_2\tau \tag{33}$$

where the admissible form of the background metric must be:

$$A=A(\theta-N\phi) \;\; , \;\; C=C(\theta-N\phi) \quad , \quad A'(0)=0 \;\; , \;\; C'(0)=0 \; , \tag{34}$$

(N – arbitrary integer), with angular frequencies $\omega_{1,2}$ in (33):

$$(\omega_1)^2=\frac{A(0)}{1+a^2/N^2} \quad , \quad \omega_2=\frac{\omega_1}{N} \; . \tag{35}$$

We conclude that the LL-brane performs a *spiral motion* in the toroidal extra dimensions with winding frequencies as in (35).

4. Lightlike Branes in Gravitational Backgrounds: Codimension-One

This case is qualitatively different from the case of codimension ≥ 2. Here the metric (24) acquires the form of a general spherically symmetric metric:

$$ds^2 = -A(t,y)(dt)^2 + C(t,y)h_{ij}(\vec{\theta})d\theta^i d\theta^j + B(t,y)(dy)^2 \tag{36}$$

where $\vec{x} \equiv \vec{\theta}$ are the angular coordinates parametrizing the sphere S^p.

The *LL-brane* equations of motion (19)–(22) now take the form:

$$-A + B\,\dot{y}^2 = 0 \;\; , \text{ i.e. } \;\; \dot{y} = \pm\sqrt{\frac{A}{B}} \quad , \quad \partial_t C + \dot{y}\,\partial_y C = 0 \tag{37}$$

$$\partial_\tau \chi + \chi \left[\partial_t \ln\sqrt{AB} \pm \frac{1}{\sqrt{AB}}\Big(\partial_y A + p\,a_0 \partial_y \ln C\Big)\right]_{y=y(\tau)} = 0 \tag{38}$$

First let us consider static spherically symmetric metrics in standard coordinates:

$$ds^2 = -A(y)(dt)^2 + A^{-1}(y)(dy)^2 + y^2 h_{ij}(\vec{\theta})d\theta^i d\theta^j \tag{39}$$

where $y \equiv r$ is the radial-like coordinate. Here we obtain:

$$\dot{y} = 0 \;\; , \;\; \text{i.e. } \; y(\tau) = y_0 = \text{const} \quad , \quad A(y_0) = 0 \; , \tag{40}$$

implying that the *LL-brane* positions itself *automatically* on the horizon y_0 of the background metric ("horizon straddling"). Further, for the dynamical tension we get:

$$\chi(\tau) = \chi_0 \exp\left\{\mp\tau\left(\partial_y A\,|_{y=y_0} + \frac{2p\,a_0}{y_0}\right)\right\} \quad , \quad \chi_0 = \text{const} \; . \tag{41}$$

Thus, we find a time-asymmetric solution for the dynamical brane tension which (upon appropriate choice of the signs ($\mp$) depending on the sign of the constant factor in the exponent on the r.h.s. of (41)) *exponentially "inflates" or "deflates"* for large times.

Next consider spherically symmetric metrics in Kruskal-Szekeres-like coordinates:

$$ds^2 = A\left(y^2 - t^2\right)\left[-(dt)^2 + (dy)^2\right] + C\left(y^2 - t^2\right)\, h_{ij}(\vec{\theta})d\theta^i d\theta^j \tag{42}$$

where (t,y) play the role of Kruskal-Szekeres's (v,u) coordinates for Schwarzschild metrics [10]. Here the *LL-brane* equations of motion yield:

$$\dot{y} = \pm 1 \;\; , \;\; \text{i.e. } \; y(\tau) = \pm\tau \quad , \quad \left(y^2 - t^2\right)|_{t=\tau, y=y(\tau)} = 0 \; , \tag{43}$$

i.e., again the *LL-brane* locates itself *automatically* on the horizon ("horizon straddling"), whereas for the dynamical tension we obtain:

$$\chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{p\, a_0\, C'(0)}{A(0)C(0)}\right\} . \tag{44}$$

Thus, we find a time-symmetric *"inflationary" or "deflationary"* solution with *quadratic* time dependence in the exponential for the dynamical brane tension (depending on the sign of the constant factor in the exponent on the r.h.s. of (44)).

Let us also consider "cosmological"-type metrics:

$$ds^2 = -(dt)^2 + S^2(t)\left[(dy)^2 + f^2(y) h_{ij}(\vec{\theta}) d\theta^i d\theta^j\right] \tag{45}$$

where $f(y) = y$, $\sin(y)$, $\sinh(y)$. The *LL-brane* equations of motion give:

$$\dot{y} = \pm \frac{1}{S(\tau)} \quad , \quad S^2(\tau)\, f^2(y(\tau)) = \frac{1}{c_0^2} \;\; , \;\; c_0 = \text{const} \; , \tag{46}$$

implying: $S(\tau) = \pm\frac{1}{c_0\, y_0}\, e^{-c_0\,\tau}$, $S(\tau) = \pm\frac{1}{c_0}\, \cosh\left(c_0(\tau+\tau_0)\right)$ or $S(\tau) = \mp\frac{1}{c_0}\, \sinh\left(c_0(\tau+\tau_0)\right)$, respectively, where $y_0, \tau_0 = \text{const}$.

For the dynamical brane tension we obtain "inflation"/"deflation" at $\tau \to \pm\infty$:

$$\chi(\tau) = \chi_0 (S(\tau))^{2p\, a_0 - 1} \quad , \quad \chi_0 = \text{const} \tag{47}$$

Example 1: de Sitter embedding space metric in Kruskal-Szekeres-like (Gibbons-Hawking [11]) coordinates. In this case:

$$ds^2 = A(y^2-t^2)\left[-(dt)^2 + (dy)^2\right] + R^2(y^2-t^2) h_{ij}(\vec{\theta}) d\theta^i d\theta^j \tag{48}$$

$$A(y^2-t^2) = \frac{4}{K(1+y^2-t^2)^2} \quad , \quad R(y^2-t^2) = \frac{1}{\sqrt{K}}\, \frac{1-(y^2-t^2)}{1+y^2-t^2} \tag{49}$$

(K is the cosmological constant).
We obtain exponential "inflation" at $\tau \to \pm\infty$ for the dynamical tension of *LL-branes* occupying de Sitter horizon:

$$\chi(\tau) = \chi_0 \exp\{\tau^2\, p\, a_0 K\} \; . \tag{50}$$

Example 2: Schwarzschild background metric in Kruskal-Szekeres coordinates [10]. In this case (here we take $D = p + 2 = 4$)

$$ds^2 = A(y^2-t^2)\left[-(dt)^2 + (dy)^2\right] + R^2(y^2-t^2) h_{ij}(\vec{\theta}) d\theta^i d\theta^j \tag{51}$$

$$A = \frac{4R_0^3}{R} \exp\left\{-\frac{R}{R_0}\right\} \quad , \quad \left(\frac{R}{R_0} - 1\right) \exp\left\{\frac{R}{R_0}\right\} = y^2 - t^2 \tag{52}$$

(here $R_0 \equiv 2G_N m$). We obtain exponential “deflation” at $\tau \to \pm\infty$ for the dynamical tension of *LL-branes* sitting on the Schwarzschild horizon:

$$\chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{a_0}{R_0^2}\right\} . \tag{53}$$

Example 3: Reissner-Nordström background metric in Kruskal-Szekeress-like coordinates. In this case (here again $D = p + 2 = 4$):

$$ds^2 = A(y^2 - t^2)\left[-(dt)^2 + (dy)^2\right] + R^2(y^2 - t^2) g_{ij}(\vec{\theta}) d\theta^i d\theta^j . \tag{54}$$

In the region around the outer Reissner-Nordström horizon $R = R_{(+)}$, *i.e.*, for $R > R_{(-)}$ ($R = R_{(-)}$ – inner Reissner-Nordström horizon), the functions $A(x)$, $R(x)$ are defined as:

$$y^2 - t^2 = \frac{R - R_{(+)}}{\left(R - R_{(-)}\right)^{R_{(-)}^2/R_{(+)}^2}} \exp\left\{R \frac{R_{(+)} - R_{(-)}}{R_{(+)}^2}\right\} \tag{55}$$

$$A(y^2 - t^2) = \frac{4R_{(+)}^4 \left(R - R_{(-)}\right)^{1 + R_{(-)}^2/R_{(+)}^2}}{\left(R_{(+)} - R_{(-)}\right)^2 R^2} \exp\left\{-R \frac{R_{(+)} - R_{(-)}}{R_{(+)}^2}\right\} \tag{56}$$

We find here *exponentially “deflating”* tension for the *LL-brane* sitting on the outer Reissner-Nordström horizon:

$$\chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{a_0}{R_{(+)}^2} \left(1 - \frac{R_{(-)}}{R_{(+)}}\right)\right\} \tag{57}$$

(a phenomenon similar to the case of LL-brane sitting on Schwarzschild horizon (53)).

In the region around the inner Reissner-Nordström horizon $R = R_{(-)}$, *i.e.*, for $R < R_{(+)}$, the functions $A(x)$, $R(x)$ are given by:

$$y^2 - t^2 = \frac{R - R_{(-)}}{\left(R - R_{(+)}\right)^{R_{(+)}^2/R_{(-)}^2}} \exp\left\{R \frac{R_{(-)} - R_{(+)}}{R_{(-)}^2}\right\} \tag{58}$$

$$A(y^2 - t^2) = \frac{4R_{(-)}^4 \left(R_{(+)} - R\right)^{1 + R_{(+)}^2/R_{(-)}^2}}{\left(R_{(-)} - R_{(+)}\right)^2 R^2} \exp\left\{-R \frac{R_{(-)} - R_{(+)}}{R_{(-)}^2}\right\} \tag{59}$$

In this case we obtain *exponentially "inflating"* tension for the *LL-brane* occupying the inner Reissner-Nordström horizon:

$$\chi(\tau) = \chi_0 \exp\left\{\tau^2 \frac{a_0}{R^2_{(-)}}\left(\frac{R_{(+)}}{R_{(-)}} - 1\right)\right\} . \tag{60}$$

The latter effect is similar to (50) – the exponential brane tension "inflation" on de Sitter horizon.

5. Lightlike Branes in Kerr Black Hole and Black String Backgrounds

Let us consider $D=4$-dimensional Kerr background metric in the standard Boyer-Lindquist coordinates (see *e.g.* [12]):

$$ds^2 = -A(dt)^2 - 2Edt\, d\varphi + \frac{\Sigma}{\Delta}(dr)^2 + \Sigma(d\theta)^2 + D\sin^2\theta(d\varphi)^2) , \tag{61}$$

$$A \equiv \frac{\Delta - a^2\sin^2\theta}{\Sigma} \quad , \quad E \equiv \frac{a\sin^2\theta\,\left(r^2+a^2-\Delta\right)}{\Sigma} \tag{62}$$

$$D \equiv \frac{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}{\Sigma} , \tag{63}$$

where $\Sigma \equiv r^2 + a^2\cos^2\theta$, $\Delta \equiv r^2 + a^2 - 2Mr$, and the following ansatz for the *LL-brane* embedding (here $p=2$):

$$X^0 \equiv t = \tau \;\; , \;\; r = r(\tau) \;\; , \;\; \theta = \sigma^1 \;\; , \;\; \varphi = \sigma^2 + \widetilde{\varphi}(\tau) . \tag{64}$$

In this case the *LL-brane* equations of motion (19)–(20) acquire the form:

$$-A + \frac{\Sigma}{\Delta}\,\dot{r}^2 + D\sin^2\theta\;\dot{\varphi}^2 - 2E\,\dot{\varphi} = 0$$

$$-E + D\sin^2\theta\;\dot{\varphi} = 0 \quad , \quad \frac{d}{d\tau}\left(D\Sigma\sin^2\theta\right) = 0 . \tag{65}$$

Inserting the ansatz (64) into (65) we obtain:

$$r = r_0 = \text{const with } \Delta(r_0) = 0 \quad , \quad \textit{i.e. } r_0 - \text{Kerr horizon} , \tag{66}$$

$$\omega \equiv \dot{\varphi} = \frac{a}{r_0^2 + a^2} - \text{constant angular velocity} . \tag{67}$$

Among the X^μ-equations of motion (22) only the X^0-equation yields additional information, namely, we obtain from the latter an exponential *"inflating"/"deflating"* solution for the dynamical *LL-brane* tension in Kerr black hole background:

$$\chi(\tau) = \chi_0 \, \exp\left\{\mp\tau\left(\frac{1}{M} - \frac{1}{r_0}\right)\right\} . \tag{68}$$

From (66)–(68) we conclude that, similarly to the spherically symmetric case, *LL-branes* moving as test branes in Kerr rotating black hole background automatically straddle the Kerr horizon and in addition they are "dragged" (rotate along) with angular velocity ω given in (67). Note that the latter expression coincides precisely with the definition of Kerr horizon's angular velocity (Eq.(6.92) in ref.[12]). Furthermore, as in the spherically symmetric case we find "mass inflation/deflation" effect on Kerr horizon via the exponential time dependence of the dynamical *LL-brane* tension.

The above analysis applies straightforwardly to the case of lightlike string ($p = 1$) moving in $D = 4$ Kerr black hole background, *i.e.*, a case of codimension two. Here the lightlike string positions itself automatically on the equator of the horizon (66) ($r = r_0$, $\theta = \frac{\pi}{2}$) and again rotates along the latter with the angular velocity (67).

Along the same lines we can analyze the dynamics of codimension-two test *LL-brane* ($p = 2$) in a $D = 5$ boosted black string background [13]. The metric of the latter reads:

$$ds^2 = -A(dt)^2 + \frac{(dr)^2}{f} + r^2\left[(d\theta)^2 + \sin^2\theta\,(d\varphi)^2\right] + B(dz)^2 - 2\mathcal{E}dt\,dz \tag{69}$$

$$A(r) \equiv 1 - (1-f(r))\cosh^2\beta \quad , \quad B(r) \equiv 1 + (1-f(r))\sinh^2\beta \; ,$$
$$\mathcal{E}(r) \equiv -(1-f(r))\sinh\beta\cosh\beta \quad , \quad f(r) \equiv 1 - \frac{r_0}{r} \; , \tag{70}$$

where β is the boost rapidity parameter, and we employ the following ansatz for the *LL-brane* embedding:

$$X^0 \equiv t = \tau \;\; , \;\; r = r(\tau) \;\; , \;\; \theta = \sigma^1 \;\; , \;\; \varphi = \sigma^2 \;\; , \;\; z = z(\tau) \; . \tag{71}$$

Inserting (71) into *LL-brane* equations of motion (19)–(22) we obtain:

$$r(\tau) = r_0 \quad , \quad z(\tau) = \omega\,\tau + z_0 \quad \text{with} \;\; \omega = -\tanh\beta \quad , \quad \chi = \text{const} \; . \tag{72}$$

In other words the codimension-two test *LL-brane* automatically occupies the sphere S^2 of the $S^2 \times S^1$ horizon of the boosted black string and winds the circle S^1 of the horizon with angular velocity ω given in (72).

6. Further Developments and Outlook

Codimension one *LL-branes* possess natural couplings to bulk Maxwell $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{p+1}}$ gauge fields ($D-1 = p+1$, see refs.[7]):

$$\widetilde{S}_{\rm LL} = \int d^{p+1}\sigma\,\Phi(\varphi)\left[-\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + L\bigl(F^2\bigr)\right]$$
$$-q\int d^{p+1}\sigma \varepsilon^{ab_1\ldots b_p} F_{b_1\ldots b_p}\partial_a X^\mu \mathcal{A}_\mu(X)$$
$$-\frac{\beta}{(p+1)!}\int d^{p+1}\sigma\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}\mathcal{A}_{\mu_1\ldots\mu_{p+1}}(X)$$

As shown in [7] by considering bulk Einstein-Maxwell+Kalb-Ramond-field system coupled to a *LL-brane*:

$$S = \int d^D x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4e^2} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{p!2} \mathcal{F}_{\mu_1 \ldots \mu_D} \mathcal{F}^{\mu_1 \ldots \mu_D} \right] + \widetilde{S}_{\rm LL} \quad (73)$$

the *LL-brane* can serve as a material and charge source for gravity and electromagnetism and, furthermore, it generates *dynamical* cosmological constant through the coupling to the Kalb-Ramond bulk field:

$$K = \frac{8\pi G_N}{p(p+1)} \beta^2 \ . \quad (74)$$

There exist the following static spherically symmetric solutions of the coupled system. The bulk space-time consists of two regions with different geometries separated by a common horizon occupied by the *LL-brane*. The matching of the metric components across the horizon reads (using the same notations as in (39)):

$$A_{(-)}(y_0) = 0 = A_{(+)}(y_0) \ , \ \left(\partial_y A_{(+)} - \partial_y A_{(-)}\right)_{y=y_0} = -\frac{16\pi G_N}{(2a_0)^{p/2-1}}\, \chi \quad (75)$$

As discussed in more details in a forthcoming paper [14], conditions (75) allow for a *non-singular* black hole type solution where the geometry of the interior region (below the horizon) is de-Sitter with *dynamically* generated cosmological constant K (74), whereas the outer region's geometry (above the horizon) is Schwarzschild or Reissner-Nordström.

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